Chapter 3

The rapid implementation of asset/liability models for sovereign risk management

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Abstract: Uncertainty makes economic and project management more difficult for any entity. This is especially true for sovereigns that have experienced substantial financial volatility and shocks in the last decade, especially those with substantial debt and commodity price exposures. Furthermore, the development of a strategic approach at the country level for the analysis of that uncertainty has lagged behind as most approaches exclude, for example, trade flows and fiscal dimensions. A World Bank research project undertook to rectify that situation. In this paper, we will focus on those aspects of that project related to the systems technology and its selection, development, and refinement for central banks and ministries of finance in developing countries. We will show how those selections make the transfer of the technology feasible and implementable in sovereign institutions.

Key words: Asset/liability models, sovereign risk management, ALM

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1. INTRODUCTION

1.1 Background

Global financial markets have been very volatile in recent decades with large fluctuations in commodity prices, foreign exchange rates, interest rates, and capital flows. Many developing countries have large exposures to these risks. They often have large external debts and considerable foreign exchange reserves, exposing them to interest and exchange rate risks. Many developing countries depend on (primary) commodity exports for generating foreign exchange, or need to rely on imports for energy and to supplement basic food supplies. Adverse movements in international commodity prices can affect them greatly. All these risks have played a role in raising the debt burdens and negatively affecting economic performance of many developing countries.

Improving asset/liability management (ALM) is more important than ever before. During the last two decades, a broader range of financial tools (credit swaps, derivatives, etc.) has become available to deal with these risks. The breadth of tactical risk management tools has expanded greatly and now includes many types of borrowings and assets, forwards, swaps, plain vanilla and exotic options, etc. ALM strategies have become more sophisticated and concepts such as value-at-risk are now commonly used. Moreover, developing countries have in recent years gained some access to risk management tools.

Yet, the development of a strategic approach for ALM at the country level has lagged behind. Typical approaches to country ALM are copied from approaches for firms and financial institutions, do not incorporate country-specific factors, and strategic interactions are missing. They often exclude, for example, trade flows and fiscal dimensions. Modeling flexibility is very limited, with country adaptation often happening through a piece-meal approach by basic analysis rather than optimization. More generally, their perspective is often the development of benchmarks that are constant over time. However, by requiring a benchmark, which is constant over time,

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they fail to incorporate the dynamic realignment of portfolios. The treatment of uncertainty is typically also very limited and constraints are typically not included in the process itself, but rather through iterating around a solution.

ALM for sovereigns in developing countries often has to consider risks on a much broader scale than well-established corporations in developed countries. It is also essential that they adopt a truly dynamic approach. Risks to include concern not only the government's own direct exposures, such as those arising from debt and reserves, but also those arising from contingent risks from the banking systems or state-owned enterprises. Approaches need to be related to measures of the government's earning potential, such as the sensitivity of fiscal revenues to global factors. Without these factors, approaches to risk ignore the existence of natural hedges in the external and fiscal sectors, limit the analysis to "on-balance" liabilities only, and ignore many important constraints. Approaches need to be dynamic: developing countries face, for example, many constraints in rapidly adjusting their assets and liabilities as transactions costs can be high. ALM strategies as pursued for corporations and applied in developing countries can thus be less than optimal and may even add to their risk.

The changing nature of reserve management risk also highlights the need for more sophisticated risk management tools. There has been impressive growth in the level of total foreign exchange reserves of central banks, of which Asia and Latin America account for almost all the increase[1]. This growth implies an opportunity cost so that central banks are considering more active investment strategies for a portion of their portfolio that has a low probability of being used for intervention purposes. The challenge then becomes how to apportion the portfolio between a liquid portfolio that could be used for intervention purposes and another for investment purposes while managing asset class risk, credit risk, currency risk, and interest rate risk. The objective becomes one of enhancing returns with sound asset and liability management and a solid public mandate while constraining risk to a suitable level.

[1] Cassard and Folkerts-Landau [2000a].

Realising the importance of strategic ALM tools for sovereigns, the World Bank undertook a research project to develop a framework for their implementation[2]. This paper is about those aspects of that project related to the selection, integration, and implementation of the systems for the technology to support the framework and how those choices facilitate the implementation of ALM models in sovereign institutions.

1.2 The research project

Existing approaches to ALM were discarded since they could not satisfy the project's requirements. Several major international banks were invited to the World Bank in 1997 to propose the technology they had for application to ALM problems in developing countries. They were all found to be lacking the ability to adequately address the issues mentioned in Section 1.1.

The research project subsequently developed a framework to address the ALM issues and solutions to the technical barriers to applying and implementing ALM in developing countries. A workshop was held at the end of the research. It was attended by 20 staff from sovereign institutions worldwide.

The research project did not have the funding for building a user interface and for further development to simplify technology transfer and implementation. That and many subsequent enhancements have been undertaken by The RisKontrol Group GmbH, Bern, Switzerland[3].

1.3 How we will proceed in this paper

We will begin by defining the problem that provided the basis for the system/language selections. We will define the mathematical framework to

[2] Research proposal by Claessens et.al. [1995].
[3] www.RisKontroller.com

address the problem and discuss examples of the modelling language implementation. We will then discuss the user interface and finally the process wherein the technology is implemented and used. We assume that the reader has a basic awareness of the systems GAMS[4] and MATLAB[5] that were used in the research project.

2. THE PROBLEM

2.1 Defining the problem

The problem is not simply a mathematical problem that can be addressed by a mathematical model. There was no intention to build a mathematical model that would be applicable to all countries, but rather a framework that could be easily and rapidly tailored to a specific country situation. The framework had to integrate easily into the risk management framework of the sovereign institution. This meant that we not only needed to build a mathematical framework but also to define a process for applying that framework within the institution. This framework and process were the critical defining attributes of the project.

2.2 Problem attributes

In order to address the issues discussed in Section 1.1, the technology had to have the following attributes:

a) be strategic in nature;
b) include risks from and interactions between debt and reserves;
c) include contingent risks such as from the banking sector;
d) be related to a government's earning potential; and

[4] See the extensive documentation at www.gams.com. GAMS was originally developed at the World Bank.
[5] See the extensive documentation at www.matlab.com.

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e) incorporate sovereign objectives and constraints.

Furthermore, its implementation had to allow for:

a) easy transport to a developing country;
b) running on a PC or a modest size platform;
c) providing insight into risk and the stochastic model;
d) a friendly graphical user interface (GUI); and
e) capabilities for users to build intuition into and confidence in the underlying processes and models.

Any technology to address the issues of Section 1.1 adequately would likely have to be complex. Therefore item e) above was very important in getting institutions to accept and use it. The technology had to allow for the eventual integration of risks in interest rates, exchange rates, liquidity, credit, actuarial, processes without a history, economic processes, commodity prices, extreme/rare events, implied prices, theories, and expert opinions.

2.3 Basic design principles of the technology

The first goal was to define the mathematical framework. The basic design principles for this framework and for the technology are that they had to:

a) incorporate many risk factors;
b) provide capabilities to shape density functions as the method to understand, reduce, and compare risks;
c) solve the mathematical models;
d) be open and understandable;
e) be easily and rapidly customised and modified;
f) provide insight into solutions; and
g) be transferable to a developing country on a PC platform.

It was the case that defining the mathematical framework and addressing a) and b) constrained by g) were the biggest challenges during the research phase. The solutions to these are discussed in Claessens et. al. [2000]. The principles c) through g) would define the systems/language choices for developing the technology.

3. THE STOCHASTIC ASSET/LIABILITY MODEL FRAMEWORK

3.1 The mathematical framework for stochastic processes

Before discussing the systems, we will define the mathematical framework for the kinds of problems we want to solve. This framework will satisfy the basic design principles of Section 2.3.

There were two classes of variables that needed to be defined. First were the stochastic variables of interest rates, exchange rates, commodity prices, macroeconomic processes, etc. We will call these the "tree variables" for reasons that will be obvious shortly. The other class was stochastic decision variables, which we will call the "decision variables". It was assumed that the decision variables would not affect the tree variables, which eliminated those problems with decisions of such a magnitude that they would affect interest rates and exchange rates, for example. These later problems are still a subject of research and were not a priority for this work.

We define a multi-factor process for generating the tree variable values using Equation 3.1 to describe how these values change over time.

$$\frac{dx_i(t)}{x_i(t)} = \mu_i(x,\bar{t})dt + \sum_{j=1}^{n} b_{ij}(x,\bar{t})\sigma_j(x,\bar{t})d\omega_j(t), \quad \forall i,j$$
$$x_i(0) = x_i^0, \quad i = 1,\ldots,m \quad \text{and} \quad t \in [0,T] \tag{3.1}$$

Where x_i^0 are given (today's rates), μ_i, b_{ij}, σ_j are constants that need to be estimated, and ω_j are independent (standard) Wiener processes. We break

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up the time interval $[0,T]$ into sub-intervals defined by $[0,t_1,\ldots,t_{l-1},T]$. We assume that μ_i, b_{ij}, σ_j are constant over any sub-interval of time and estimate the values separately in each sub-interval. The use of the notation $\bar{t}$ in Equation 3.1 means that the values for μ_i, b_{ij}, σ_j can change but are constant over the sub-intervals.

We do not discuss the process of estimating μ_i, b_{ij}, σ_j here but note that we allow their values to be determined by a combination of the following:

a) stochastic estimation based upon history;
b) implied values (e.g. from current derivative values);
c) theoretical values (e.g. satisfying uncovered interest parity); and
d) expert views.

We use econometric estimation in the case of macroeconomic data such as fiscal deficit, trade, and GDP and integrate these processes with Equation 3.1. Alternatively, we use 3.1 with stochastic volatility since it has been shown that certain GARCH processes converge in distribution to diffusions[6].

Equation 3.1 has exact stochastic solutions[7] given by Equations 3.2. These are used to construct the tree variable values.

$$\begin{aligned} &E\{x(t)\} = \left(x_1^0 e^{\mu_1 t}, x_2^0 e^{\mu_2 t}, \ldots, x_n^0 e^{\mu_n t}\right), \\ &\operatorname{cov}\{x_l(t), x_k(t)\} = E\{x_l(t)\} E\{x_k(t)\} \left[\exp\left(t\sum_{j=1}^{n} b_{lj} b_{kj} \sigma_j^2\right) - 1\right] \\ &\operatorname{var}\{x_l(t)\} = E\{x_l(t)\}^2 \left[\exp\left(t\sum_{j=1}^{n} b_{lj}^2 \sigma_j^2\right) - 1\right] \end{aligned} \tag{3.2}$$

One specific realization or instance of variable values over time, called a scenario or sample path, is given by Equation 3.3. We use this equation to generate scenarios for various reasons including comparing them to trees,

[6] For this and for a general discussion of continuous-time methods, see Sundaresan [2000].
[7] See Claessens et. al. [1995].

testing trees, for terminal branches of a tree, and for building intuition. However, we do not use Equation 3.3 for building the tree.

$$x_i(t) = x_i^0 \exp\left[\left(\mu_i - \frac{1}{2}\sum_{j=1}^{n}\sigma_j^2 b_{ij}^2\right)t + \sum_{j=1}^{n}\sigma_j b_{ij}\omega_j(t)\right] \tag{3.3}$$

We also generate variable values on the tree that are functions of other tree variables such as stochastic cash flows, extreme/rare events, and for other variables. An example of the later is variables based upon level such as low levels of GDP triggering contingent liabilities in state-owned enterprises.

3.2 The mathematical framework for the model

Our model framework is a dynamic stochastic programming model used to obtain values of decisions variables. We take the view that the way to compare solutions to dynamic stochastic programming models is to compare the density functions[8] of optimal decision variables and of tree variables. These variables may be objective functions or other decision variables or even tree variables or some stochastic function of these. There may be more than just one density function that is compared. One example of this is a central bank that wants to maximize its return on its reserve portfolio, wants a very low probability of getting negative returns in any year, and wants to keep as passive a portfolio as possible. In order to compare solutions we would want to see the density function of absolute returns, the density function of percentage returns, and the density functions of portfolio activity; all over time.

The approach to controlling risk then becomes one of shaping or sculpting the density functions of stochastic functions of the variables. There are several ways to do that including:

[8] For the method of computing density functions see Wets [1998].

a) Appraisal functions[9] to maximize the probability of portfolio returns being within a specified interval;
b) "Squishing" density functions so that their shape is narrower, which represents a less risky outcome;
c) Conditional value-at-risk[10] contstraints;
d) Using natural hedges or adding derivatives[11];
e) Minimizing value-at-risk (VaR);
f) Constraining downside risks; and
g) Combinations and other tailored methods.

We discuss the appraisal function here. A simple example of an appraisal function we could use for an asset portfolio is given in Equation 3.4. This equation represents the maximization of the expected value of the discounted (δ) appraisal function (Θ see Equation 3.5) of wealth (W). The parameters r_1, r_2 control the slope of the graph of the function and the maximization process tries to "push" as much of the probability mass into the interval $[p,q]$. An example of how this is used is by a central bank that wants very strongly to get a return on its reserves portfolio of at least 5% but is not concerned, because of political reasons, to get a return above 6.5%. Hence $[p,q]=[.05,.065]$.

$$\text{MAX} \quad E\left[\sum_{t\in T}\delta^t r_1^t\left(q^t-p^t\right)\Theta_{\frac{r_1^t}{r_2^t}}\left(\frac{W^t-p^t}{q^t-p^t}\right)\right] \tag{3.4}$$

[9] Claessens et. al. [1998].
[10] See Uryasev [2000].
[11] See Blejer and Schumacher [2000] for a discussion of derivatives and contingent liabilities in central banks.

$$\Theta_e(\lambda) = \begin{cases} \lambda & \text{for } \lambda \le 0 \\ \lambda - \dfrac{\lambda^2(e-1)}{2e} & \text{for } 0 \le \lambda \le 1 \\ \dfrac{\lambda}{e} + \dfrac{e-1}{2e} & \text{for } \lambda \ge 1 \end{cases} \tag{3.5}$$

The function Θ is linear below p^t and above q^t and quadratic on the interval $[p^t, q^t]$. At the end points, the function values and the first derivatives match.

Another example of an appraisal function is given in Equation 3.6. This function represents the minimization of the expected value of the appraisal function (Θ see Equation 3.7) of the ratio of total external debt (TB) to GDP (Ω).

$$\text{MIN } E\left\{ r^T\left(q^T - p^T\right)\Theta\left[\frac{\dfrac{TB^T}{\Omega^T} - p^T}{q^T - p^T}\right]\right\} \tag{3.6}$$

$$\Theta(\lambda) = \begin{cases} 0 & \text{if } \lambda \le 0 \\ \dfrac{\lambda^2}{2} & \text{if } 0 \le \lambda \le 1 \\ \lambda - \dfrac{1}{2} & \text{if } \lambda \ge 1 \end{cases} \tag{3.7}$$

The minimization "pushes" the probability mass of debt/GDP as much as possible below q and is rather indifferent to "pushing" it below p. There are several other ratios (or indicators) that any specific country might wish to use that can be constructed in similar ways to the above. These can be found in detail in the IMF report on "Debt- and Reserve-Related Indicators of External Vulnerability", 2000. They could include ratio of reserves to short-term external debt, ratio of reserves to imports, external debt over exports, and share of foreign currency external debt in total external debt. Another ALM objective example is the total terminal public wealth and private savings as a ratio to GDP.

The model variables consist of asset and liability holdings today and at various rebalancing dates in the future along with other decision variables such as derivative purchases, policy decisions, etc. The future decision variables are stochastic, of course.

The model constraints, in addition to stochastic constraints like conditional value-at-risk, that might be imposed are many and varied and we only list some possibilities here:

a) legal limits on asset classes;
b) portfolio rollover constraints;
c) transaction cost limits;
d) cash flow requirements;
e) currency transfer constraints;
f) market access constraints;
g) liquidity constraints; and
h) other policy constraints.

4. THE COMPUTER SYSTEMS IMPLEMENTATION

4.1 The systems/language choices

There were three separate and distinct areas that required specific systems/language choices in the development of the technology:

a) the tree generation procedure;
b) the stochastic optimization model formulation and solution; and
c) graphics generation and user interfaces.

There are several possible language choices for optimization and for graphical user interfaces. Several considerations went into our choices in addition to the requirements of Section 2. It is not our intention to write a comparative paper here and therefore we will mention some of the positive reasons for choosing GAMS and MATLAB. We do however feel that these are the best choices for this class of problems.

From Section 3 we see that we needed a model formulation language that satisfies not only our principles of design but also handles our mathematical framework. The later requires a large-scale nonlinear programming solver, or at least a large-scale linear-quadratic programming solver. It is the General Algebraic Modeling System (GAMS) with the CONOPT solver that handles these problems[12] very nicely. Furthermore, GAMS/CONOPT is open and understandable and runs on many computers including PCs. In order for the model formulation to be easily and rapidly customised, some additions to the language were required to handle stochastic problems. These are discussed in Section 4.2.3.

[12] Models with a piecewise linear-quadratic objective and up to 100,000 variables were solved on a PC in about ½ an hour. This number of variables is small for a typical dynamic stochastic programming problem. However because of the tree generation method, a very large problem is often not required.

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MATLAB has the ability to rapidly generate GUIs, has excellent graphical capabilities, can run C and C++ code, allows developing executables, links to Excel, links to other main database management systems, links to real-time financial data feeds, has financial and statistical toolboxes, and is widely used in finance. In short, it has the capabilities that we need.

4.2 The GAMS model

4.2.1 The separation of the stochastics and the dynamic stochastic programming model

In order to be easily and rapidly customised and modified, it was essential that the tree generation and the dynamic stochastic programming model formulation be completely separated. This means that the stochastic processes could be modified and different trees generated without having to change the representation of the model. Alternatively, the model could be modified, new objectives added, and constraints changed without changing the representation of the tree.

4.2.2 Building the tree

The tree structure is built up event[13] by event using Equations 3.2 by solving the moment matching problem[14]. The problem is a nonlinear programming problem that matches Equations 3.2 and satisfies other specialized constraints and objectives. Using this technique, we can keep the number of branches small.

This is very important since these trees do not grow exponentially but remain relatively small. For example, a four stage tree of 15 tree variables

[13] We use the word "event" here to represent a realization of values of the tree variables at some future time. The word "node" is also used in the literature.
[14] See Dupačová et al. [2000] for a discussion of this problem.

and a modest stochastic model generated a problem of under 100,000 variables. This would not be possible with the typical binary tree, which would give 2^{60} events at the horizon.

In order to implement this technique, we developed the general code in GAMS to sweep through the tree solving the problem at each event. This can be coded very nicely in GAMS. In addition, we also trap and rerun any moment matching problem that fails since we do not want to have the entire tree generation fail at the 800th event, for example. We test for non-optimality or infeasibility and make some adjustments to the moment matching problem so that it solves. This has become very reliable. The trapping technique was very important, as the moment matching problem is highly nonlinear since we also allow the probability values to be variable allowing much wider degrees of freedom. An example of a small tree depicting one of the tree variables is given in Figure 4.1. One must be careful in interpreting this tree, as the branches do not have equal probability. Another graph is used to depict this by looking head-on into the branches at any given time and making the size of the dot depicting a branch proportional to its probability mass.

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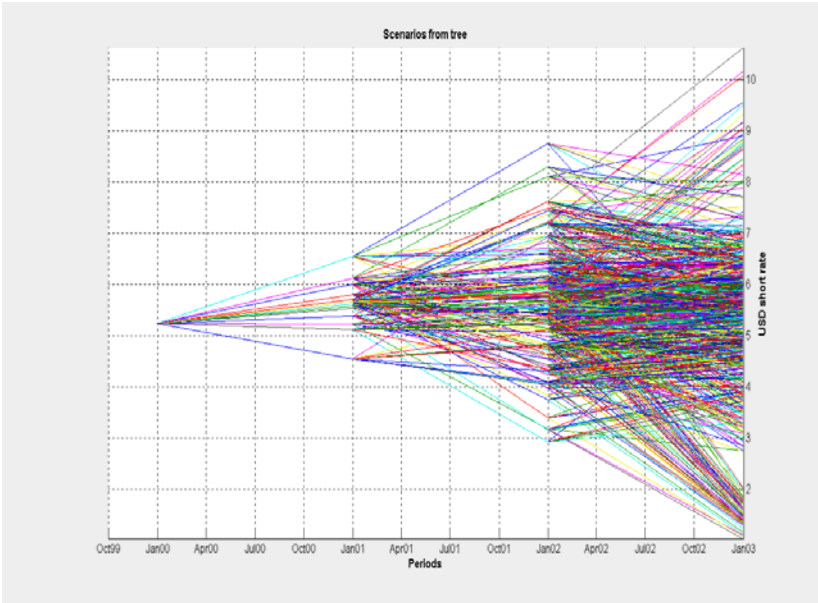

Figure 4.1. Tree generation depicting one variable.

In the example of Figure 4.1, there are three stages where a sequence of moment matching problems need to be solved. In the first stage there are 15 different possible realizations. Within the GAMS program we solve a moment matching problem to compute those values and assign them to a variable. In this case we assign them to the GAMS variable **rho(I,C,T,EV)**. This represents the interest rate of return for asset **I** in currency **C** at time **T** and event **EV**.

4.2.3 Building the model

Once the tree variables are computed, we use those values in the dynamic stochastic programming model but we formulate the GAMS representation of the model without any knowledge of the tree structure.

In order to do this, we need to know two things:

1. What are all the events that can occur at any time, **T**.
2. Given an event, **EV**, at time **T**, what are all the antecedent events to **EV**.

This information is captured in two GAMS set mappings; **XST(T,EV)** and **ANTE(T,EV,TT,EVV)**. The first gives the pairs of set elements indicating an event **EV** exists in time **T**. The second provides all the pairs **(TT,EVV)** that occur prior to **(T,EV)** on the same scenario or sample path. These two mappings describe the tree structure completely. Although the mapping **XST** contains redundant information, it makes the model formulation very easy. Similarly for the **ANTE** array, we would not need both **T** and **EV** but having them makes the model formulation very easy.

The GAMS code for the Equations 3.4 and 3.5 is given in Figure 4.2 in order to illustrate the ease of describing these complex equations.

The function is linear below p and above q and quadratic in between with the function values and first derivatives matching at the ends. Since the function is non-decreasing and we are maximizing, we can break it up into three segments as is done here. We only define the equation at all the events of the last time period. The rest of the formulation should be clear, except for one addition to Equation 3.4.

We include dividing the total wealth by the variable **basket(T,EV)**. We have computed the total wealth in US dollars but wish to measure it in a different numeraire, which is called the currency basket. This tree variable is the exchange rate of US dollars to the currency basket. The currency basket is a proportion of different currencies chosen to match the currency composition of imports. This can be a desirable measure for central banks in some cases since the implication is that holding the currency basket composition is a currency risk free position for the country. The exchange rate for the basket is computed from the exchange rates of the tree variables. This notation makes it easy to change the objective function measure to values in any currency or currency basket.

```
*-----------------------------------------------------------
*   Variable segmentation for appraisal function

 seg1(XST(T,EV))$TLAST(T)..
         TW(T,EV)/basket(T,EV) - plev(T)
    =E=  (qlev(T) - plev(T))
        *( SG1(T,EV) + SG2(T,EV) + SG3(T,EV) )
 ;
*-----------------------------------------------------------
*   Bounds for segmentation

 SG1.UP(XST(T,EV))$TSET(T) = 0 ;
 SG2.UP(XST(T,EV))$TSET(T) = 1 ;
 SG2.LO(XST(T,EV))$TSET(T) = 0 ;
 SG3.LO(XST(T,EV))$TSET(T) = 0 ;
*-----------------------------------------------------------
*   Appraisal function for objective

 obj..   FAPP  =E= SUM(XST(T,EV)$TSET(T),
                   prob(T,EV)*delta(T,EV)*r1(T)
                             *(qlev(T) - plev(T))
              *(
                  SG1(T,EV)
                + SG2(T,EV)
                - SQR(SG2(T,EV))*((r1(T)/r2(T)) - 1)
                                *r2(T)/(2*r1(T))
                + SG3(T,EV)*r2(T)/r1(T)
               )
         )
  .
```

Figure 4.2. GAMS code for Equation 3.4 and 3.5

Figure 4.3 gives the density functions of the optimal objective of Equations 3.4 and 3.5 solved for a specific problem[15]. This first one is obtained using $(r_1, r_2) = (1, .5)$ and the second one using $(r_1, r_2) = (100, .1)$. The exact values are not so important but rather to note that in the second case much of the probability mass is pushed into the interval defined by $[p^t, q^t] \approx [1.4, 1.6] \times 10^4 = (\text{starting assets}) \times [1.05^3, 1.065^3]$.

Another example of GAMS code is given in Figure 4.4, which is the formulation of Equations 3.6 and 3.7. The variable **scgdp** is used to scale the GDP values for p and q. The variable **macro("GDP",T,EV)** contains the tree variable values for GDP. The rest of the formulation should be clear.

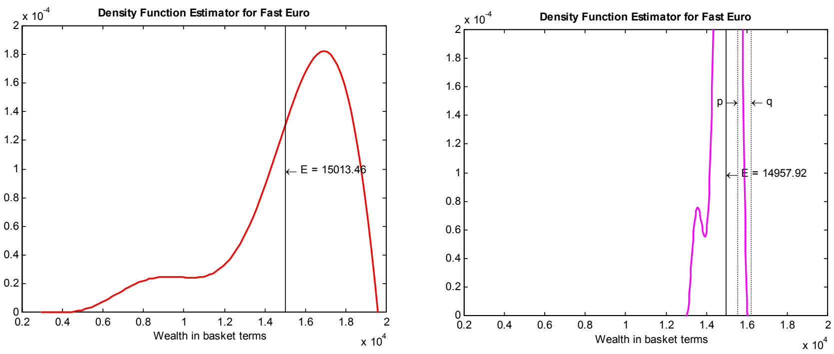

Figure 4.3: Comparison of two densities

[15] See Claessens et al. [2000] for a detailed discussion of that problem.

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```
*---------------------------------------------------------
-
*   Variable segmentation for appraisal function

 seg1(XST(T,EV))$TLAST(T)..
         TB(T,EV)/(macro("GDP",T,EV)) - scgdp*pgdp(T)
    =E=  scgdp*(qgdp(T) - pgdp(T))
               *( SG1(T,EV) + SG2(T,EV) + SG3(T,EV) )
 ;
*---------------------------------------------------------
-
*   Bounds for segmentation

 SG1.UP(XST(T,EV))$TLAST(T) = 0 ;
 SG2.UP(XST(T,EV))$TLAST(T) = 1 ;
 SG2.LO(XST(T,EV))$TLAST(T) = 0 ;
 SG3.LO(XST(T,EV))$TLAST(T) = 0 ;
*---------------------------------------------------------
-
*   Appraisal function for objective

 obj..   FAPP  =E=   SUM(XST(T,EV)$TLAST(T),
                       prob(T,EV)*r(T)*(qgdp(T) - pgdp(T))
             *(
                  (SG2(T,EV)**2)/2
                 + SG3(T,EV)
```

Figure 4.4. GAMS code for Equations 3.6 and 3.7.

In this framework, modifications to the objective functions are very easy to make and we do not need to take account of the tree structure in any way. Furthermore, if we introduce new tree variables that are functions of existing tree variables (such as the currency basket), we can compute these values on the tree trivially.

Of course, if we introduce new variables on the tree, we may need to return to compute the entire tree anew using the equations 3.1 and 3.2. However, we need not change our objective function representation or any other model equation in GAMS.

We discuss one last snippet of GAMS code in Figure 4.5 that uses the antecedent mapping. The **netflow(C,XST(T,EV))** constraint defines cashflows by currency **C** and for each event **EV** that exists in time **T**. We do not include all the code for that constraint formulation as it is too long. Figure 4.5 illustrates the formulation for interest return on assets (fixed rate is assumed here) and assets sold. The variable **rho(I,C,TT,EVV)** gives the values of the interest rate of return of asset **I** in currency **C** acquired at time **TT** at event **EVV**. The reader should see how this formulation can be converted to a variable rate definition.

```
*------------------------------------------------------------
*    Material balance equation for funds flows
netflow(C,XST(T,EV))..
   ...

+ SUM(I,
     SUM(ANTE(T,EV,TT,EVV)$((ORD(T) - ORD(TT)) LE mat(I) ),
             rho(I,C,TT,EVV)*
                 SUM(EVVV$ANTE(T,EV,T-1,EVVV),
                     A(I,C,TT,EVV,T-1,EVVV))
     )
     )
*    interest on assets
+ SUM(I,
       SUM(ANTE(T,EV,TT,EVV)$((ORD(T) - ORD(TT)) LT mat(I)
),
       SUM(EVVV$ANTE(T,EV,T-1,EVVV),
           tcs(I,C)*eta(I,C,TT,T,EV)*(
               A(I,C,TT,EVV,T-1,EVVV) - A(I,C,TT,EVV,T,EV)
                                 )
       )
       )
     )
   *   sales of assets
*------------------------------------------------------------
-
```

Figure 4.5. Snippet of GAMS code using **ANTE** mapping.

The first summation is over the set of events that are antecedents of the event **(T,EV)** that do not exceed the maturity of the asset **I**. The decision variable **A(I,C,TT,EVV,T-1,EVVV)** gives the holdings of asset **I** in currency **C** acquired at event **(T-1,EVVV)** and held at event **(TT,EVV).** The summation **SUM(EVVV$ANTE(T,EV,T-1,EVVV),** merely has the effect of picking the correct event **EVVV** in the previous time period, **T-1**.

This multi-indexed approach to asset holdings was first implemented in a dynamic stochastic programming model developed by Lane and Kreuser [1980] at the Treasurer's Department of the World Bank and used for strategic planning on a weekly basis for their fixed-income portfolio of $US 6 billion in 1977. This is one of the first commercial applications of multi-period optimization models under uncertainty used on a regular basis for financial strategic planning. Other applications are described in Ziemba and Mulvey [1998].

The second use of the **ANTE** mapping in Figure 4.5 formulates the sale of assets. The variable **tcs(I,C)** is that portion of the asset price remaining from the asset sales after accounting for transaction costs and other fees. The tree variable **eta(I,C,TT,T,EV)** is the price percentage gain or loss of asset **I** in currency **C** acquired at time **TT** and priced at time **T** and event **EV**. The expression **A(I,C,TT,EVV,T-1,EVVV) - A(I,C,TT,EVV,T,EV)** gives the change in holdings from time **T-1** to time **T** or equivalently, the amount sold in time **T**.

Users of GAMS may be concerned that these mappings will significantly increase model generation time. However, in our experience so far the model generation time is less than the model solution time so the issue is not a critical one. It may become important for models with a million or more variables. A representative example for a problem generating about 100,000 variables and solved using CONOPT is model solution time about 30 minutes and model generation time about 15 minutes on a 200MHz portable. However, the benefits of having an easily modified representation far outweigh this cost since, for this class of models, changes are likely to occur frequently.

5. THE GUI AND THE GRAPHICS

With so many variables and complex variable structures, graphical capabilities must be available in order to provide understanding, insight, and intuition into the model and its results. It is also necessary to provide a user interface to guide the user in applying the technology. Both of these requirements are met using the system MATLAB.

The models require a model formulation language and a very powerful optimizer and that is provided by GAMS and CONOPT and other solvers within GAMS. Moreover, the links between GAMS and MATLAB exist or are not difficult to develop.

Michael Ferris [1998] has automated the process of exchanging variable values between the two. We also want to change the model code in GAMS depending upon user selections in the user interface. This is done by generating snippets of GAMS code for output from MATLAB and included in GAMS via the $include statement.

McGoldrick [2000] provides an excellent discussion on the rapid development of financial applications with emphasis on MATLAB. He discusses the process of planning, developing, and distributing a typical financial application and provides some comparison with other development environments.

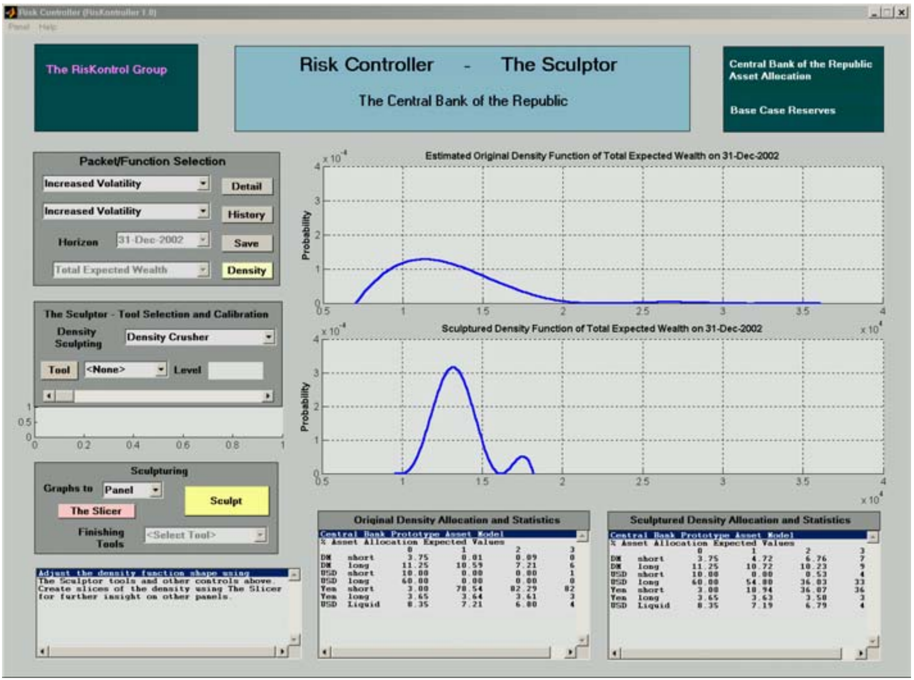

Figure 5.1. *Panel for comparing density functions by shaping them.*

Figure 5.1 illustrates the panel developed using MATLAB for comparing density functions by shaping them using the various techniques described in Section 3.1. The first density function is of the optimal value of the currency basket of reserves for a central bank. The second density function is the result of applying the "Density Crusher" to squish the density plus an application of a constraint on negative returns on the portfolio. The impact of these, besides reshaping the density, is to shorten the duration of the composition of reserves. This density reshaping process proceeds by restarting the model from the previous optimal solution. We generate the

GAMS code necessary for the restarted model depending on user selections in the GUI, output it to a file that can be included in a GAMS restart file, and run the model. This makes the entire process simple and efficient and most important, easy to update. Furthermore, it is entirely controlled from the MATLAB GUI. In this framework it is easy to modify the model, GUI, graphical outputs, etc. in order to tailor them to individual country needs very rapidly.

6. INTEGRATING INTO THE INSTITUTION

Simply having the tools necessary for analysis is no guarantee that they will be used correctly. They must be accepted, understood, and integrated into the institutional risk management framework[16].

The user interface and graphical capabilities go a long way in supporting acceptance and understanding. Furthermore, the technology can be explained by an extension to an already known technology, VaR[17]. But, most importantly, the technology was designed based upon how it would be integrated into the institutional risk management framework and the functions that are undertaken in that framework. We discuss that in this section.

The technology is strategic in nature and as such is expected to be used by a Middle Office or a unit with similar functions[18]. For example, in a central bank the technology might be used to analyze or create a benchmark for currency composition and reserves level. In a ministry of finance it might be used to analyze and determine characteristics of new debt. In a ministry of planning it might be used to analyze and help determine the

[16] See IMF draft guidelines [2001] for a discussion of a risk management framework in a central bank, IMF and World Bank [2001] for a risk management framework in a ministry of finance, and Cassard and Folkerts-Landau [2000b] for ALM frameworks in central banks and ministries of finance.

[17] See Jorion [2000] for a discussion on VaR.

[18] See Goldman Sachs [1998] for a general discussion of these functions.

currency of choice for a World Bank loan. Or it might be used at the sovereign level for ALM analysis.

The functions that are characteristic of this institutional risk management framework include benchmark creation, assumption comparisons, policy comparisons, stress testing, and model validation including backtesting, sensitivity analysis, and model risk assessment.

A beginning point for the analysis of any of these functions requires a baseline model solution with a minimum of special assumptions. This baseline model we call the "market neutral" formulation. This formulation is characterised by:

a) Expected values of interest rates are computed so that they satisfy statistically the interest rates implied in the term structure. We apply this principle to the current rates as well as to any future rates by calibrating the expected interest rates in the future to the forward rates implied today.
b) Volatility and correlations are computed from implied values where possible. Otherwise, we compute the volatility of and correlations among interest and exchange rates and other factors from historical information. We do this separately for each time interval into the future.
c) Going forward, we allow time-varying volatility to reflect the possibility of regimes over which the uncertainty is more or less stable and to incorporate the mean-reversion present in many asset prices. We calibrate the stochastic processes separately over the short and long term.
d) In terms of exchange rates, expected values are computed such that the Uncovered Interest Parity hypothesis holds, i.e., exchange rates are expected to appreciate or depreciate by the differential in interest rates.
e) Other assumptions to invoke price neutrality.

These assumptions or similar ones reflect the view that the risk manager does not take a position beyond what the market is telling him. This is easy to formulate in our mathematical framework and using GAMS it is easy to implement.

Having obtained a solution to this problem, the risk manager may want to take an expert view or he may want to impose an expert view on the model given to him by members of senior management, the investment committee, or by the board of directors. We can thusly compare solutions and we call this an assumption comparison. A policy comparison may consist of a policy to constrain the use of certain asset classes or to impose some other policy such as duration limits if that was used in the past. Or it may be a more complicated constraint such as a very low probability of getting negative returns. The policy comparison is made with and without the constraint. The idea is to analyze and compare the alternative assumptions and policies.

In order to do this we shape the density functions to get results appropriate to the institutional goals and objectives, which have been articulated in their broadest sense by the board of directors. It is important to note that these goals and objectives will change from time to time depending on external circumstances. Furthermore, there are generally many objectives and therefore multiple density functions to be shaped simultaneously. Once solutions are obtained for each assumption or policy, techniques of Section 3.2 are applied to shape the density functions appropriately.

Measurement and comparison of solutions in our framework is based upon density functions of tree and decision variables or some function of them. The discrete cumulative distribution is easily extracted from a given model formulation. Continuous distributions and densities are obtained using an implementation of the method of Wets [1998] using GAMS/CONOPT.

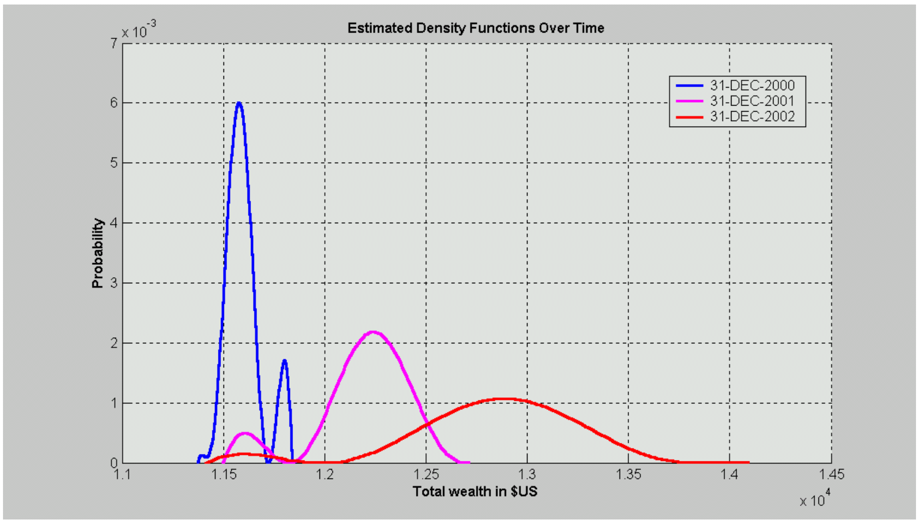

Figure 6.1: Density functions of wealth

Figure 6.1 illustrates the density functions over time of the total wealth of the reserves portfolio of the Central Bank of the Republic of Colombia[19] projected under policy constraints for the market neutral or baseline case.

Figure 6.2 illustrates the density functions over time for the corresponding percent of the portfolio composition in dollars. This gives an indication of how passive the portfolio will be under varying conditions. A portfolio with narrower density functions here is considered good because it does not require as much active currency management.

There may also be other considerations. Figure 6.3 gives a comparison of two sets of density functions whereby a policy comparison is made of the consideration whether or not to constrain no-negative returns to as small a probability as possible. For a bond portfolio, this just reduces the duration.

[19] See Claessens et. al. [2000] for a detailed discussion of the case of Colombia.

For a complete comparison we must also compare the portfolio return and other factors.

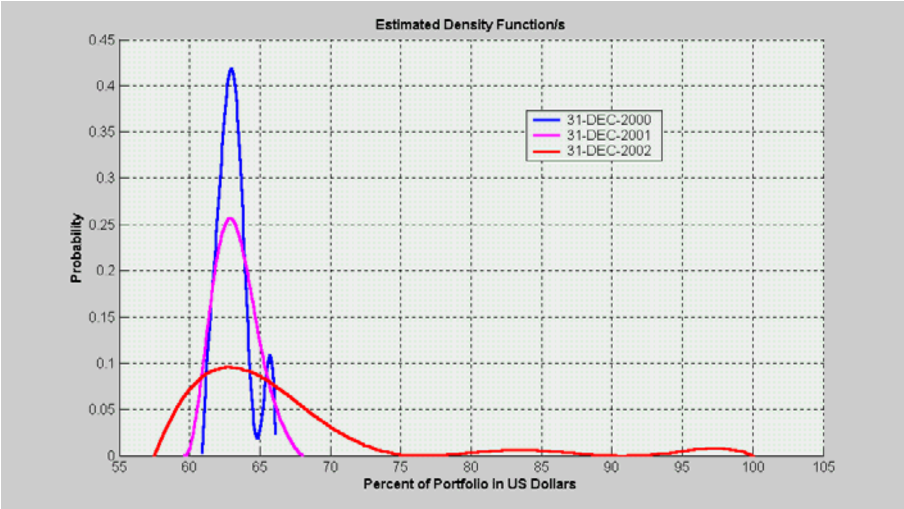

Figure 6.2: Percent portfolio composition in dollars

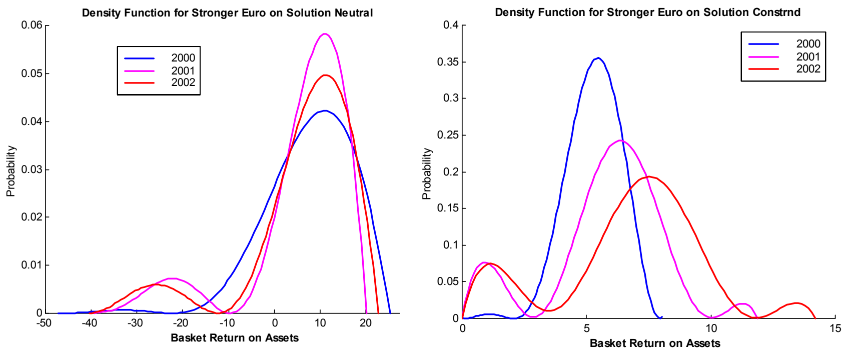

Figure 6.3: Basket returns with and without non-negative returns

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There are many other density functions that can be analysed, but we only mention one more of particular interest. That is given in Figure 6.4 and illustrates the net loss densities over time of picking one policy over another. The interesting aspect of these graphs is that the densities are fairly symmetrical about the origin. This means the policy choices do not make much difference in terms of the distribution of total return.

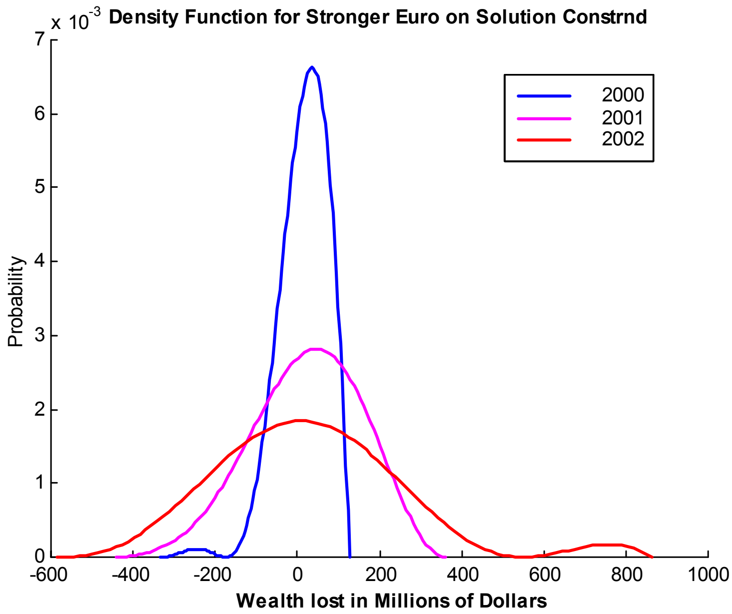

Figure 6.4: Net loss of choosing one policy over another

The process of benchmark creation is one that fits very nicely into our framework. This is because benchmarks can be created that address integrated risks such as combining interest rate and exchange rate risks for a central bank, debt and GDP for a ministry, and trade flows with fiscal deficit, reserves, and debt for an ALM approach.

The process of stress testing fits nicely into our framework also. Because of the way trees are generated, stress scenarios can be easily added at any event and depend on the level of other variables. Alternatively, stress events can be easily added at any given time. Expected values, volatility, or correlations can be easily overridden with expert views but still maintain other important relationships. Correlations can easily be modified according to tree variable values or made to converge at specified times of stress. Additional constraints can be added such as market access constraints for example as the model formulation is easy to change in GAMS.

Model validation is an easy process to undertake in this framework, however, a discussion of this is beyond the scope of this paper.

7. SUMMARY

The success of an institutional risk management framework depends on several factors. First and foremost, there must be an institutional commitment from the board of directors on down. Following that, it is clear from many experiences with debt and reserves management that a proper institutional framework is key. This needs to involve the availability of good personnel, data, reporting and accounting procedures, and accountability. Furthermore, proper ALM requires a very careful analysis of the real, underlying objectives and constraints. Often such an analysis will indicate underlying problems, which need to be fixed first before any ALM can be undertaken. And the analysis, as well as institutional weaknesses might well suggest that the best way forward is to use simple models to analyze specific areas of concerns.

Once the basics are in place, the framework discussed in this paper shows how one could begin to make more informed decisions under uncertainty for debt and asset management. The process is rigorous and the structured framework provides significant gains in analysis and insight. One needs to and is able to investigate the whole spectrum of distributions of asset returns, exchange rate returns, and their effects on uncertainty of wealth, currency composition, maturity, and asset composition in order to understand the reasons for their volatility or lack thereof. While many have used simpler measures—such as value-at-risk, standard deviations, partitions of the densities, or shortfalls—these are only partial measures and less informative than the full density measures.

Sovereign ALM has special requirements not usually incorporated into typical ALM approaches to firms and financial institutions. These requirements can be met in a very general overall framework such as discussed in this paper. This overall framework consists of the mathematical framework, the systems framework, and the institutional risk management framework. These require a large-scale modeling and solution capability, stochastic process generation capabilities, a user interface with graphics capabilities and tools for their rapid development and modification. These system/language capabilities are critical for a successful implementation. And they need to be integrated into the risk management framework within the financial institution.

These system/language capabilities have been implemented in a technology using MATLAB for a GUI and graphics capability and using GAMS for formulating and solving large-scale nonlinear programming problems and for generating trees and scenarios from stochastic processes.

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